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HEAT TRANSFER BY LAMINAR FILM CONDENSATION ON SPHERE SURFACES

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NOMENCLATURE

A, area [m^2];
c_p, heat capacity [J/kg K];
D, diameter of sphere [m];
g, gravitational acceleration [m/s^2];
h, heat-transfer coefficient [W/m^2 K];
$h'_{fg} = h_{fg} + 0{\cdot}68\, c_p(T_s - T_w)$, latent heat of vaporization corrected to account sensible heat of subcooling in the film after [5] [J/kg];
H, height of cylindrical part of vessel [m];
k, thermal conductivity [W/m K];
$\dot{m}$, mass rate of flow [kg/s];
Nu, Nusselt number;
Pr, Prandtl number;
$\dot{Q}$, heat flux [W];
R, radius of sphere surface [m];
$T_s - T_w$, difference between saturation temperature and wall temperature [K];
u, velocity in x direction [m/s];
x, coordinate measuring distance along circumference from the upper stagnation point of sphere [m];
y, coordinate measuring radial distance outward from sphere surface [m];
δ, thickness of the condensate film [m];
Θ, angular coordinate [rad];
μ, dynamic viscosity [kg/m s];
ρ, ρ_v, density of condensate and density of vapor [kg/m^3].

Subscripts
H, hemisphere;
0, initial conditions;
S, sphere;
($\bar{\ }$), average value.

INTRODUCTION

IN CHEMICAL apparatus and devices of food industry, condensation processes very often occur on sphere surfaces. Mixer evaporators to condense vegetable or fruit pulps are an example of such devices.

The first results of calculations of heat transfer at film condensation on the sphere were given by Dhir and Lienhard [1]. They have used Nusselt's theory and have developed the general expression for the heat transfer coefficient on plane and axisymmetric bodies in nonuniform gravity. But this expression, obtained with the initial condition $\delta_0(x = 0) = 0$ for blunt bodies (such as the sphere, where $\delta_0 \neq 0$) formally is not valid. Recently Yang [2] has presented the results of numerical solution of momentum and energy equations, describing a thin layer of condensate in the form of laminar film running downward over the sphere. This communication presents the results of Nusselt's model analysis of heat transfer by laminar film condensation on sphere surfaces taking into account liquid wetting.

ANALYSIS

According to Nusselt's treatment [3] the downward flow of the condensate in the film, under the action of the gravity force, describes the balance equation between the gravity tangential component and the viscous forces $(\delta - y)(\rho - \rho_v)g \sin\Theta = \mu(\mathrm{d}u/\mathrm{d}y)$, acting on the liquid element of volume $2\pi R \sin\Theta(\delta - y)R\,\mathrm{d}\Theta$. The expression for the velocity distribution is

$$u(y) = (\rho - \rho_v)\mu^{-1} g \sin\Theta(\delta y - 0{\cdot}5y^2)$$

and the mass flow of condensate in the film through the section at the given angle Θ is:

$$\dot{m} = \int_0^\delta \rho u(y) 2\pi R \sin\Theta\,\mathrm{d}y = \tfrac{2}{3}\pi\rho(\rho - \rho_v)\mu^{-1} g \sin^2\Theta R\delta^3. \quad (1)$$

As the flow of condensate proceeds from Θ to $\Theta + \mathrm{d}\Theta$, the film thickness varies from δ to $\delta + \mathrm{d}\delta$ as a result of both the influx of additional condensate and the change of the ring section area. This additional influx of condensate is

$$\mathrm{d}\dot{m} = 2\pi\rho(\rho - \rho_v)\mu^{-1} g \sin\Theta R \times (\tfrac{2}{3}\delta^3 \cos\Theta\,\mathrm{d}\Theta + \sin\Theta\delta^2\,\mathrm{d}\delta). \quad (2)$$

The heat flux removed by the element of the sphere surface $\mathrm{d}A = 2\pi R^2 \sin\Theta\,\mathrm{d}\Theta$ must be equal to the incremental mass flow of condensate times the latent heat of condensation of the vapor:

$$\mathrm{d}Q = k\frac{T_s - T_w}{\delta}\mathrm{d}A = \mathrm{d}\dot{m}h'_{fg} \quad (3)$$

hence after substituting equation (2) we obtain

$$(a\delta^4 \cos\Theta - b)\,\mathrm{d}\Theta + \delta^3 \sin\Theta\,\mathrm{d}\delta = 0 \quad (4)$$

where: $a = 2/3$, $b = \mu(T_s - T_w)Rk/\rho(\rho - \rho_v)gh'_{fg}$. This equation can be reduced to the complete differential equation. Therefore the solution of this equation in the form of $\delta = \delta(\Theta)$ can be obtained from the relation

$$\tfrac{2}{3}\delta^4 \int_{\Theta_0}^{\Theta} (\sin\Theta)^{5/3} \cos\Theta\,\mathrm{d}\Theta - b\int_{\Theta_0}^{\Theta} (\sin\Theta)^{5/3}\,\mathrm{d}\Theta + \tfrac{1}{4}(\sin\Theta_0)^{8/3}(\delta^4 - \delta_0^4) = 0$$

hence

$$\delta^4 = b\frac{\int_{\Theta_0}^{\Theta} (\sin\Theta)^{5/3}\,\mathrm{d}\Theta + C}{\tfrac{1}{4}(\sin\Theta)^{8/3}}, \text{ where } C = \tfrac{1}{4}\frac{\delta_0^4}{b}(\sin\Theta_0)^{8/3}. \quad (5)$$

Let us consider two cases of the laminar film condensation on sphere and bottom hemisphere surfaces.

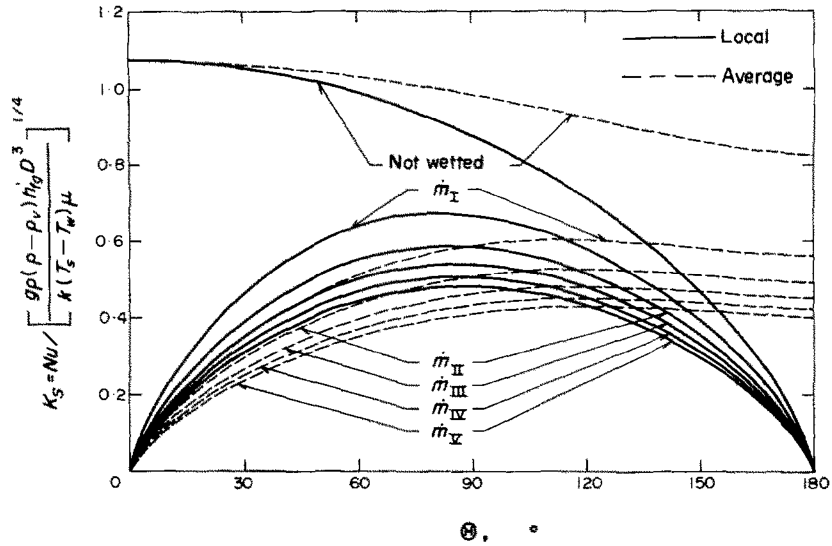

FIG. 1. Distributions of heat-transfer coefficients on a sphere surface.

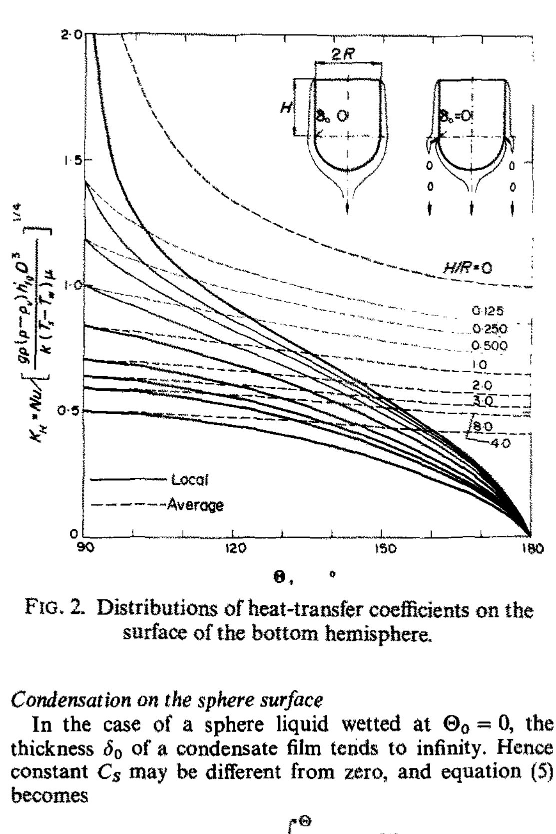

FIG. 2. Distributions of heat-transfer coefficients on the surface of the bottom hemisphere.

Condensation on the sphere surface

In the case of a sphere liquid wetted at $\Theta_0 = 0$, the thickness δ_0 of a condensate film tends to infinity. Hence constant C_S may be different from zero, and equation (5) becomes

$$\delta_S^4(\Theta, C_S) = bY_S, \text{ where } Y_S = \frac{\int_0^{\Theta} (\sin\Theta)^{5/3}\,d\Theta + C_S}{\frac{1}{4}(\sin\Theta)^{8/3}} \quad (6)$$

If the amount of liquid wetting the sphere at $\Theta_0 = 0$ per time unit is $\dot{m}_{S-0}$, then from equations (1) and (6) it results, that

$$C_S = \frac{1}{4b}\left[\frac{\dot{m}_{S-0}\mu}{\frac{2}{3}\pi\rho(\rho-\rho_v)gR}\right]^{4/3}. \quad (7)$$

In the cause of a simple sphere (sphere I on Fig. 3), not wetted the thickness of condensate film is determined. Hence constant is $C_{S-I} = 0$. If the amount of liquid which drops sphere II equals the amount of condensate flowing down from sphere I, then $(\dot{m}_I)_{\Theta=\pi} = (\dot{m}_{II})_{\Theta=0}$ and after utilizing equations (1) and (6) we obtain that

$$C_{S-II} = \int_0^{\pi} (\sin\Theta)^{5/3}\,d\Theta = 1{\cdot}6826.$$

Progressing further on as above, we can find that for sphere „i" constant C_{S-i} is

$$C_{S-i} = 1{\cdot}6826(i-1). \quad (8)$$

Condensation on the bottom hemisphere surface

When the condensation of vapor takes place on the surface of the bottom hemisphere then $\Theta_0 = \pi/2$, and from equation (5) we have

$$\delta_H^4(\Theta, C_H) = bY_H \quad (9)$$

$$\text{where } Y_H = \frac{\int_{\pi/2}^{\Theta} (\sin\Theta)^{5/3}\,d\Theta + C_H}{\frac{1}{4}(\sin\Theta)^{8/3}} \text{ and } C_H = \frac{1}{4}\frac{\delta_0^4}{b}.$$

If the condensation process starts at $\Theta_0 = \pi/2$, then $\delta_0 = 0$ and $C_H = 0$. Vapor heated cylindrical vessels with hemispherical bottoms are very often met with (Fig. 2). And if the amount of condensate flowing from the cylindrical part of the vessel is $\dot{m}_{H-0}$ then from equations (1) and (9) it results that constant C_H equals

$$C_H = \frac{1}{4b}\left[\frac{\dot{m}_{H-0}\mu}{\frac{2}{3}\pi\rho(\rho-\rho_v)gR}\right]^{4/3} \quad (10)$$

Whereas if local thickness of the flowing down laminar condensate film at $\Theta_0 = \pi/2$ is given as δ_0, then from the known relation for the film thickness for the vertical wall we have $C_H = H/R$.

Heat transfer

The local heat-transfer coefficient defined by formula $dQ = h(T_s - T_w)\,dA$ and equation (3) is $h = k/\delta$. After substituting δ from equations (6) or (9) we obtain the local heat-transfer coefficient in the dimensionless form as a Nusselt number.

$$Nu = \frac{hD}{k} = \frac{D}{\delta} = K\left[\frac{\rho(\rho-\rho_v)gh'_{fg}D^3}{(T_s-T_w)\mu k}\right]^{1/4} \quad (11)$$

where $K = (2/Y)^{1/4}$. The average Nusselt number over isothermal sphere surfaces is

$$\overline{Nu} = \overline{K}\left[\frac{\rho(\rho-\rho_v)gh'_{fg}D^3}{(T_s-T_w)\mu k}\right]^{1/4} \quad (12)$$

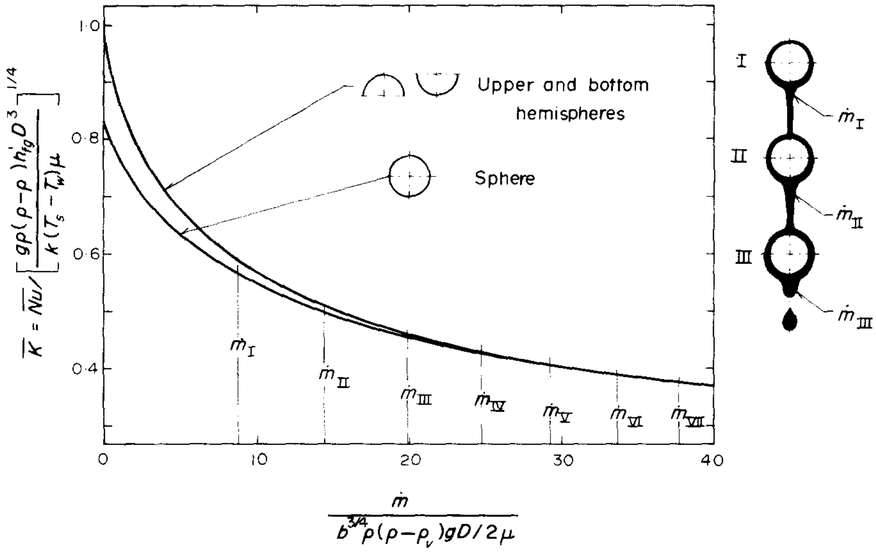

FIG. 3. Influence of liquid wetting on the average heat-transfer coefficients on sphere surfaces.

Table 1. Coefficients $\bar{K}$ in formula: $\overline{Nu} = \bar{K}\left[\frac{\rho(\rho-\rho_v)gh'_{fg}D^3}{(T_s-T_w)\mu k}\right]^{1/4}$

Authors of calculation	Sphere			Hemisphere			
	In upper stagnation point at $\Theta_0 = 0$	Average		Upper		Bottom	
				Average		Average	
		On circumference	On surface	On circumference	On surface	On circumference	On surface
Dhir and Lienhard [1]	—	0·785	—	—	—	—	—
Yang [2]	1·098	0·803	—	1·032	—	—	—
This communication	1·075	0·785	0·826	1·008	0·985	0·790	1·00

where coefficients $\bar{K}$ are calculated as averages over sphere ($\Theta_0 = 0$) or bottom hemisphere ($\Theta_0 = \pi/2$) surfaces:

$$\bar{K} = \int_{\Theta_0}^{\Theta} K \sin\Theta\, d\Theta/(\cos\Theta_0 - \cos\Theta).$$

RESULTS

The comparison of calculated coefficients $\bar{K}$ by formula (12) (without liquid wetting) with the results of previous papers gives Table. The distributions of local and average heat-transfer ratios for the sphere surface are shown in Fig. 1. On a not wetted sphere in the upper stagnation point we deal with the finite film thickness, which determines the heat-transfer coefficient. At first its value decreases slowly and then over the bottom hemisphere it decreases faster and faster as it approaches the bottom stagnation point, in which it reaches zero. It is the result of faster increase of the condensate film thickness, caused as well by vapor condensation as the decrease of the circumference and of the condense flow down and also of the gravity force tangential component. The distributions of local heat-transfer coefficients over the wetted sphere are distinguished by maximum. The distribution of local and average heat-transfer ratios over the surface of the bottom hemisphere are shown in Fig. 2. The condensate flowing down from the cylindrical part of the vessel diminishes the average heat-transfer coefficients over the hemispherical bottom of the vessel not less than 35 per cent at $H = R$ and 42 per cent at $H = 2R$. Therefore even at lower heights of the cylindrical part of the vessel, it is worth while to remove the condensate (e.g. with a little roof shown in Fig. 2) to increase the calorific effect of vessels heated by condensing vapor. Figure 3 presents the influence of the amount of liquid wetting the sphere, as well as the upper and bottom hemispheres on the average heat transfer. The average heat-transfer coefficients on the upper and bottom hemisphere surfaces are very close.

The range of conditions is not known, in which on the sphere surfaces the laminar film condensation takes place yet. But it can be safely assumed in the large range, that over the upper portion of the single sphere the condensation process is laminar and surface tensions may be neglected, like for horizontal tubes [4]. The surface tensions, which were not taken into account in our paper, can disturb the continuous dripping of condensate from the sphere at $\Theta = \pi$. However in the case of sphere surfaces these disturbances will occur sensible later on than on the horizontal cylindrical surfaces.

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